

On Spatio-temporal Dependence of Bird Migration on Earth's Magnetic Field, Solar, and Lunar Positional Changes Across the American Region

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March 3, 2026

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Motivation

- Investigate migratory paths of six bird species in the Americas.
- Analyze spatio-temporal patterns and directional shifts.
- Examine influences of Sun, Moon, and Earth's magnetic field.
- Use advanced statistical models (Von Mises, circular-circular regression).
- Provide insights into ecological and navigational dynamics.

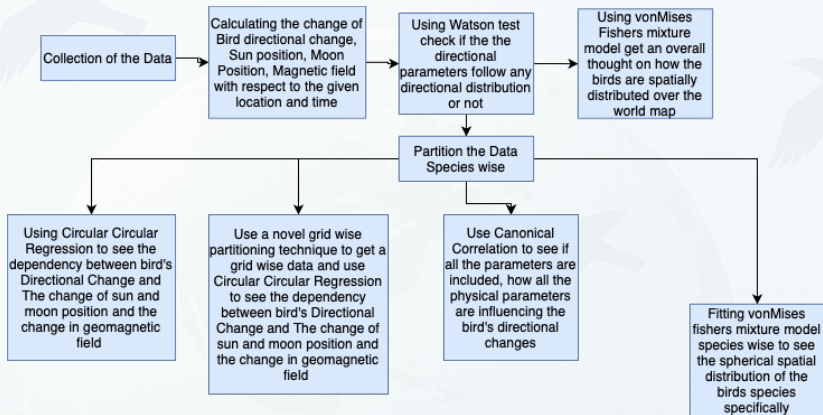


Figure 1: Flow Chart of the workflow

Why Study Migration Drivers?

- Billions of birds traverse >15000km annually – navigation still partially unexplained.
- Geomagnetic and celestial cues hypothesised since the 1960s; quantitative evidence remained sparse.
- Our contribution: **directional statistics + rich telemetry data** ⇒ rigorous, multi-factor assessment.

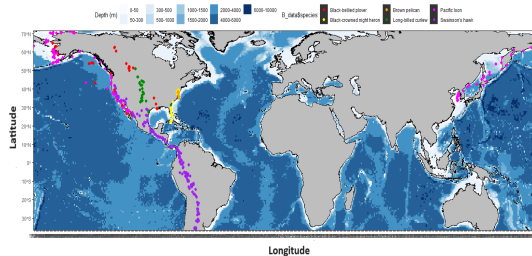


Illustration: Global flyways (BirdLife Int.). The six focused species in this study are:

- Swainson's Hawk
- Black-crowned Night Heron
- Brown Pelican
- Black-bellied Plover
- Pacific Loon
- Long-billed Curlew

Why Use Circular/Spherical Statistics?

- **Directional Movement:** Migration data involves compass directions (e.g., 0° to 360°).
- **Periodicity:** Circular data wraps around— 0° [?] 360° —linear models fail here.
- **Earth is Spherical:** Long-distance travel spans the globe—requires spherical geometry.
- **Great Circle Routes:** Birds follow shortest paths on the sphere.
- **Magnetic Navigation:** Birds respond to Earth's magnetic field—naturally spherical.
- **Sun/Moon Cues:** Celestial cues are angular, supporting circular models.
- **Appropriate Distributions:** von Mises and Fisher distributions are suited to angular data.

	Number of Observed Points	Longitude Range	Latitude Range
Swainson's Hawk	138	[−122, −61]	[−36, 42]
Pacific Loon	110	[−236, −111]	[28, 71]
Long-billed Curlew	93	[−107, −81]	[27, 51]
Brown Pelican	64	[−76, −75]	[35, 39]
Black-crowned Night Heron	85	[−82, −75]	[22, 40]
Black-bellied Plover	130	[−166, −91]	[26, 71]

- Location and Time when the birds are observed (Longitude, Latitude and Time)
- Temporal metadata → derive Sun/Moon azimuth & zenith via the formulas.
- IGRF 13 model (World Magnetic Model) for magnetic inclination/declination.
- Direction change θ computed with Haversine bearing.

Methods and feature addition for Modeling Bird Migration Patterns

- Key Techniques:
 - Haversine formula: Directional change (θ) on Earth's surface.
 - World Magnetic Model (WMM): Geomagnetic field (B_m).
 - Spatio-temporal calculations: Sun/Moon positions, zenith (Z), and azimuth (γ_s).
- Application: Model flight direction and spatial distribution across species.

Haversine Formula for Directional Change

- Computes directional change (θ) between two points on a sphere.
- Essential for navigation; accurate for small angles and distances.
- Formula (Todhunter, 1863):

$$\cos \theta = \sin \phi_A \sin \phi_B + \cos \phi_A \cos \phi_B \cos \Delta L$$

where:

- ϕ_A, ϕ_B : Latitudes of points A and B.
 - L_A, L_B : Longitudes, $\Delta L = L_A - L_B$.
- Used to derive column θ in dataset.

World Magnetic Model (WMM)

- Models Earth's geomagnetic field (B_m) using spherical harmonics.
- Magnetic field: $B_m(\lambda, \phi', r, t) = -\nabla V$, where:

$$V(\lambda, \phi', r, t) = a \sum_{n=1}^N \sum_{m=0}^M \left(\frac{a}{r}\right)^{n+1} [g_n^m(t) \cos(m\lambda) + h_n^m(t) \sin(m\lambda)] \check{p}_n^m(\sin \phi') \quad (1)$$

- Parameters:
 - $N = 36$: Truncation level.
 - $a = 6371200$ m: Geomagnetic reference radius.
 - $g_n^m(t), h_n^m(t)$: Time-dependent Gauss coefficients.
- Adjusts for crustal bias: $B_m = -\nabla V + O$.

Spatio-Temporal Sun and Moon Positions

- Computes Sun/Moon positions for migration cues.
- Subsolar point coordinates:

$$\phi_s = \delta,$$

$$\lambda_s = -15(T_{\text{GMT}} - 12 + E_{\text{min}}/60).$$

- Unit vector S to Sun's center (ECEF system):

$$S_x = \cos \phi_s \sin(\lambda_s - \lambda_0),$$

$$S_y = \cos \phi_0 \sin \phi_s - \sin \phi_0 \cos \phi_s \cos(\lambda_s - \lambda_0),$$

$$S_z = \cos \phi_0 \sin \phi_s - \cos \phi_0 \cos \phi_s \cos(\lambda_s - \lambda_0).$$

- Assumes infinite Earth-Sun distance (parallax neglected).

Zenith and Azimuth Angle Calculations

- Tracks Sun/Moon positions via zenith (Z) and azimuth (γ_s) angles.
- Zenith angle:

$$Z = \cos^{-1} S_z$$

- Azimuth angle (South-Clockwise convention):

$$\gamma_s = \arctan 2(-S_x, -S_y)$$

- Application:
 - Sorted dataset by latitude, computed Sun position differences.
 - Derived columns Z and γ_s for each species.
- Significance: Celestial cues (Sun/Moon) guide bird navigation.

Methodology: Analyzing Bird Migration Patterns using Statistical methods

- **Key Methods:**
 - Spherical mixture models (von Mises-Fisher).
 - Circular-circular regression.
 - Canonical correlation analysis.
 - Grid-based partitioning.
- **Parameters:** Bird direction (θ), solar and lunar positions, magnetic field changes.

Circular-Circular Regression

- **Purpose:** Relate bird flight direction (β) to environmental angles (α).
- **Approach:** Use complex numbers for directional data.
- **Model:**

$$E(\exp i\beta|\alpha) = \rho(\alpha) \exp(i\mu(\alpha)) = g_1(\alpha) + ig_2(\alpha)$$

$$\hat{\beta} = \arctan \left(\frac{g_2(\alpha)}{g_1(\alpha)} \right)$$

- **Approximation:** Trigonometric polynomials for $g_1(\alpha)$, $g_2(\alpha)$ [1].
- **Parameters:** Sun/moon azimuth, zenith, magnetic inclination/declination.
- **Correlation Test:** $\rho_c \neq 0$ (hypothesis testing).

- **Mixture Model:** von Mises-Fisher for spatial distribution [3, 2] [1, 4].

$$f(\boldsymbol{\theta}) = \sum_{i=1}^K \phi_i F_{\text{VMF}}(x|\theta_i)$$

- **Parameters:** Weights (ϕ_i), means (μ_i), covariances (κ_i).
- **Grid-Based Method**(Also shown in the figures below.):
 - Partition data into 10–30 grids per species.
 - Compute circular median for direction, location, and environmental factors.
 - Reduces complexity, enhances visualization.
- **Benefits:** Mitigates autocorrelation, improves interpretability.

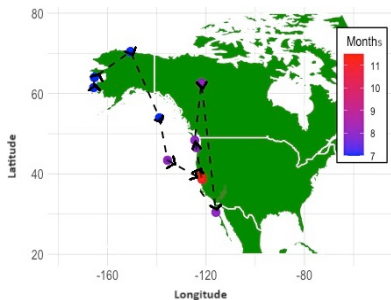
Canonical Correlation Analysis

- **Goal:** Explore relationships between environmental factors (X) and bird direction changes (Y).
- **Method:** Find linear combinations $U = Xa$, $V = Yb$ to maximize correlation:

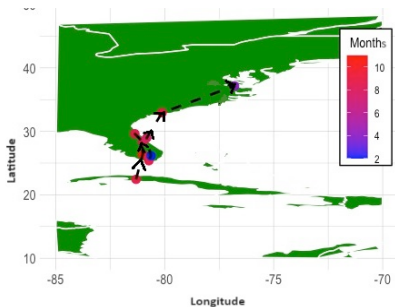
$$\rho(U, V) = \frac{\text{cov}(U, V)}{\sqrt{\text{var}(U) \cdot \text{var}(V)}}$$

- **Variables:**
 - X : Magnetic, solar, lunar changes.
 - Y : Bird direction changes (θ).
- **Output:** Canonical correlation coefficients (r_1, r_2, \dots, r_k).
- **Advantage:** Captures collective influence of multiple factors.

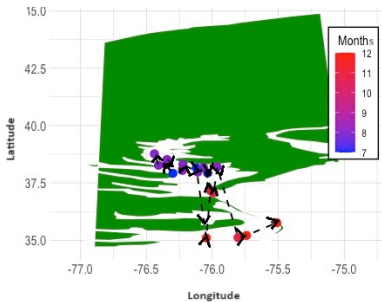
Joint analysis reveals intricate dependencies in migration patterns.



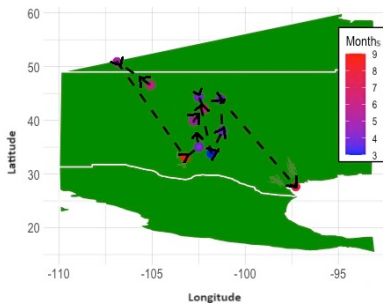
(a) For the species *Black-bellied plover*, we generated random sample datasets from the grid with 10 partitions to get an overview of the bird direction; we showed the red arrow indicating the bird direction, and the Orange dots are the point.



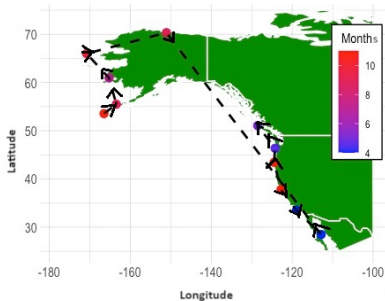
(b) For the species *Black-crowned night heron* we generated a random sample of 10 to get an overview of the bird direction we showed the red arrow indicating the bird direction and the Orange dots are the point, which is plotted over the world map.



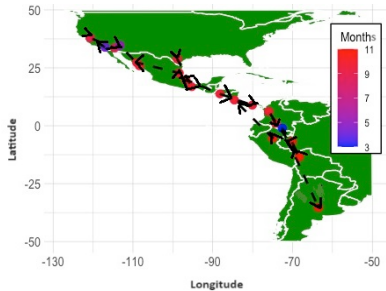
(a) For the species *Brown Pelican* we generated a random sample of 10 to get an overview of the bird direction we showed the red arrow indicating the bird direction and the Orange dots are the point, which is plotted over the world map using ggplot.



(b) For the species *Long Billed Curlew*, we generated a random sample of 10 data to get an overview of the bird direction we showed the red arrow indicating the bird direction, and the Orange dots are the point, which is plotted over the world map.



(a) For the species *Pacific Loon*, we generated a random sample of the dataset. To get an overview of the bird direction, we showed the red arrow indicating the bird direction, and the orange dots are the point, which is plotted over the world map.



(b) For the species *Swainson's Hawk*, we generated a random sample of the dataset to get an overview of the bird direction. We showed the red arrow indicating the bird's direction, and the orange dots are the point, which is plotted over the world map.

Results for Overall Bird Migration Patterns

- **Dataset Partitioning:**
 - Heuristic scatterplot revealed multimodal distribution due to bird flocking behavior.
 - Dataset partitioned into 6 clusters to achieve unimodal distributions.
- **Goodness of Fit Testing:**
 - Watson test applied to assess directional and location distributions.
- **Key Findings:**
 - North American partitions fit either Von Mises or Circular Uniform distributions.
 - South American and Alaska regions best fit Von Mises distribution.
 - Results summarized in Table 1 and visualized in Figure 5 (Density Plot).

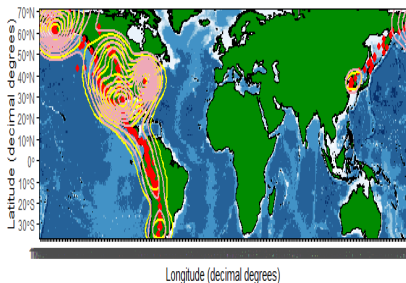


Figure 5: Density plot of the bird dataset for the American part where we used Von Mises Fishers algorithm. Where the red dots are observed bird data points, the pink color indicates the Von Mises Fishers Contour and the yellow color indicates the density plot of the dataset [5].

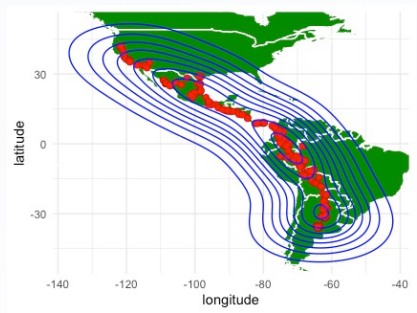
Results for Overall Bird Migration Patterns (Cont.)

K	BIC Values	K	BIC Values
1	-2700.392	11	-3561.971
2	-3234.192	12	-3566.762
3	-3398.578	13	-3540.655
4	-3440.240	14	-3544.801
5	-3461.325	15	-3525.539
6	-3637.471	16	-3519.325
7	-3609.032	17	-3491.873
8	-3594.971	18	-3488.005
9	-3577.902	19	-3464.621
10	-3401.628	20	-3504.006

Table 1: Table containing the Bayesian Information Criteria Score for Each K value. The optimal number of partitions is 6.

Results for *Swainson's Hawk* Migration Patterns

- **Flight Direction Analysis:**
 - Flight direction follows a von Mises Distribution.
 - Regression (Table 4): Magnetic field change most significant .
 - Solar Azimuth change: second most significant. .
- **Spatial Distribution:**
 - von Mises-Fisher mixture model (Table 2), where the optimal number of mixtures is 3.
- **Correlation Analysis:**
 - Geomagnetic changes: $\rho = 0.26$ (Table 3).
 - Solar position changes: $\rho = 0.232$. (Table 3)
- **Canonical correlation** (direction vs. physical parameters): 0.27006.



von Mises-Fisher density plot over observed positions of *Swainson's Hawk*.

vonMises Fisher's Mixture model & Gridwise Partition based analysis for Swainson's Hawk

K	BIC Values	K	BIC Values
1	-563.6	6	-625.0477
2	-613.82	7	-620.4164
3	-652.94	8	-601.6789
4	-646.1603	9	-593.9517
5	-638.8709	10	-610.7275

Table 2: Table containing the Bayesian Information Criteria Score for Each K value. From that, seven partitions can combine Fisher Von Mises Distribution for the location parameter(Longitude and Latitude). We checked ten values of k from 1 to 10, where the optimal number of partitions is 3. This will give us the weights(ϕ) or α values as 0.430, 0.148, and 0.4125. This is done for the bird species *Swainson's Hawk*

Parameter	ρ Value
Solar Azimuth Change	0.2329224
Magnetic declination	0.2604017
Solar Zenith Change	0.2209601
Lunar Azimuth Change	0.2159096

Table 3: *gridwise*_{swainson'sHawk}

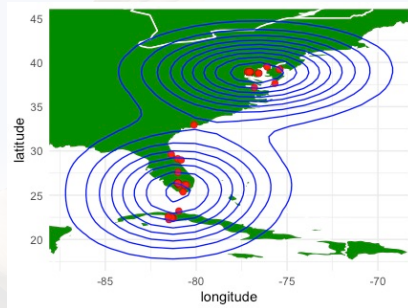
vonMises Fisher's Partitions Regression Results for Swainson's Hawk

Part	Parameters used for regression with θ	ρ values	P-value
Part-1	Magnetic declination	0.299	0.26
	Solar Azimuth Change	0.229	0.246
	Solar Zenith Change	0.242	0.64
	Lunar Azimuth Change	0.225	0.39
Part-2	Magnetic declination	0.75930	0.189
	Solar Azimuth Change	0.7316	0.272
	Solar Zenith Change	0.42	0.560
	Lunar Azimuth Change	0.71	0.291
Part-3	Magnetic declination	0.759	0.189
	Solar Azimuth Change	0.73	0.272
	Solar Zenith Change	0.424	0.5603
	Lunar Azimuth Change	0.712	0.4213

Table 4: Based on our supervised partitions where all the parameters follow von Mises distribution, we performed the circular regression and the correlation test in the above table.

Results for *Black Crowned Night Heron* Migration Patterns

- **Flight Direction Analysis:**
 - Flight direction follows a von Mises Distribution.
 - Regression (Table 7): Magnetic field change most significant .
 - Lunar Azimuth change: second most significant. .
- **Spatial Distribution:**
 - von Mises-Fisher mixture model (Table 5), optimal number of mixtures is 5.
- **Correlation Analysis:**
 - Lunar position change: $\rho = 0.60$ (Table 6).
 - Magnetic field change: $\rho = 0.53$ (Table 6).
- **Canonical correlation** (direction vs. physical parameters): 0.3089066.



von Mises-Fisher density plot over observed positions of *Black Crowned Night Heron*.

vonMises Fisher's Mixture model & Gridwise Partition based analysis for Black Crowned Night Heron

K	BIC Values	K	BIC Values
1	-257.7290	6	-432.4833
2	-341.3141	7	-423.0890
3	-360.4782	8	-437.2051
4	-394.1849	9	-427.2747
5	-442.8772	10	-409.4395

Table 5: Table containing the Bayesian Information Criteria Score for Each K value. From that, seven partitions can combine Fisher Von Mises Distribution for the location parameter(Longitude and Latitude). We checked ten values of k from 1 to 10, where the optimal number of partitions is 5.

Parameter	ρ Value
Solar Azimuth Change	0.4256448
Magnetic declination	0.5393819
Solar Zenith Change	0.2419564
Lunar Azimuth Change	0.6092881

Table 6: *gridwise*_{BlackCrownedNightHeron}

vonMises Fisher's Partitions Regression Results Black Crowned Night Heron

Part	Parameters used for regression with θ	ρ values	P-value
Part-1	Magnetic declination	0.5049	0.504
	Solar Azimuth Change	0.564	0.530
	Solar Zenith Change	0.4698	0.398
	Lunar Azimuth Change	0.5957	0.235
Part-2	Magnetic declination	0.7717	0.20545
	Solar Azimuth Change	0.342	0.4687
	Solar Zenith Change	0.682	0.660
	Lunar Azimuth Change	0.887	0.04881
Part-3	Magnetic declination	0.565	0.2557
	Solar Azimuth Change	0.498	0.322
	Solar Zenith Change	0.414	0.1525
	Lunar Azimuth Change	0.7255	0.1929
Part-4	Magnetic declination	0.6665	0.20256
	Solar Azimuth Change	0.6325	0.2108
	Solar Zenith Change	0.5539	0.9548
	Lunar Azimuth Change	0.5435	0.1731
Part-5	Magnetic declination	0.5927	0.10358
	Solar Azimuth Change	0.4927	0.7261
	Solar Zenith Change	0.5194	0.7234
	Lunar Azimuth Change	0.5369	0.3608

Table 7: Based on our supervised partitions where all the parameters follow von Mises distribution, we performed the circular circular regression and the correlation test in the above table for *Black Crowned Night Heron*

Results for *Brown Pelican* Migration Patterns

- **Flight Direction Analysis:**

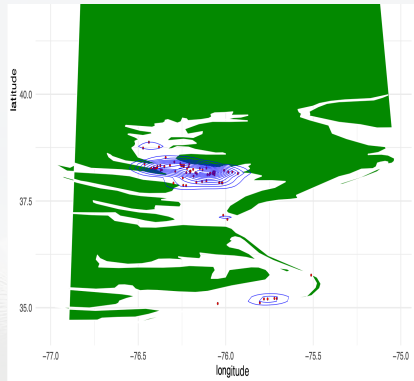
- Flight direction follows a von Mises Distribution.
- Regression (Table 10): Magnetic field change most significant .
- Solar azimuth change: second most significant. .

- **Spatial Distribution:**

- von Mises-Fisher mixture model (Table 8), optimal number of mixtures is 4.

- **Correlation Analysis:**

- Geomagnetic changes: $\rho = 0.35$ (Table 9).
- Solar azimuth change: $\rho = 0.32$ (Table 9).
- **Canonical correlation** (direction vs. physical parameters): 0.8038726.
- Inverse relationship: solar, lunar, magnetic changes.



von Mises-Fisher density plot over observed positions of *Brown Pelican*.

vonMises Fisher's Mixture model & Gridwise Partition based analysis for Brown Pelican

K	BIC Values	K	BIC Values
1	-776.002	6	-944.846
2	-932.995	7	-942.795
3	-942.365	8	-936.181
4	-945.017	9	-925.996
5	-935.657	10	-910.717

Table 8: Table containing the Bayesian Information Criteria Score for Each K value. From that, seven partitions can combine Fisher Von Mises Distribution for the location parameter(Longitude and Latitude). We checked ten values of k from 1 to 10, where the optimal number of partitions is 4. This is done for the bird species *Brown Pelican*

Parameter	ρ Value
Solar Azimuth Change	0.326116
Magnetic declination	0.3565675
Solar Zenith Change	0.2476157
Lunar Azimuth Change	0.3190035

Table 9: *gridwise*_{BrownPelican}

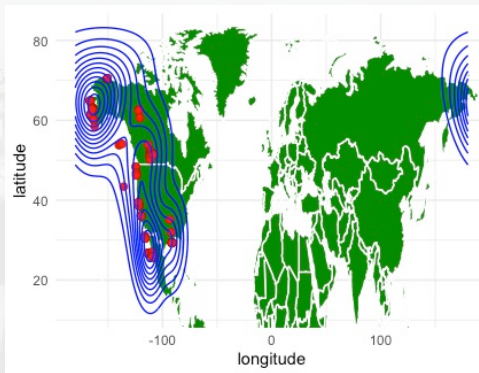
vonMises Fisher's Partitions Regression Results Brown Pelican

Part	Parameters used for regression with θ	ρ values	P-value
Part-1	Magnetic declination	0.82177	0.0386
	Solar Azimuth Change	0.5239	0.081
	Solar Zenith Change	0.3254	0.5455
	Lunar Azimuth Change	0.3254	0.0184
Part-2	Magnetic declination	0.635	0.1539
	Solar Azimuth Change	0.4337	0.3186
	Solar Zenith Change	0.4222	0.9929
	Lunar Azimuth Change	0.486	0.3319
Part-3	Magnetic declination	0.4229	0.1073
	Solar Azimuth Change	0.4032	0.11331
	Solar Zenith Change	0.2806	0.1783
	Lunar Azimuth Change	0.4088	0.1207
Part-4	Magnetic declination	0.6865	0.13256
	Solar Azimuth Change	0.6325	0.02108
	Solar Zenith Change	0.5539	0.9548
	Lunar Azimuth Change	0.5435	0.0731

Table 10: Based on our supervised partitions where all the parameters follow von Mises distribution, we performed the circular circular regression and the correlation test in the above table for *Brown Pelican*

Results for *Black Bellied Plover* Migration Patterns

- **Flight Direction Analysis:**
 - Flight direction follows a von Mises Distribution.
 - Regression (Table 13): Solar zenith position change most significant .
 - Magnetic field change: second most significant. .
- **Spatial Distribution:**
 - von Mises-Fisher mixture model (Table 11), optimal number of mixtures is 6.
- **Correlation Analysis:**
 - Solar zenith change: $\rho = 0.44$ (Table 12).
 - **Canonical correlation** (direction vs. physical parameters): 0.198874.
 - Inverse relationship: solar, lunar, magnetic changes.



von Mises-Fisher density plot over observed positions of *Black Bellied Plover*.

vonMises Fisher's Mixture model & Gridwise Partition based analysis for Black Bellied Plover

K	BIC Values	K	BIC Values
1	-731.4458	6	-1000.2251
2	-850.7185	7	-969.6409
3	-844.6696	8	-985.7637
4	-917.6586	9	-962.1987
5	-954.7474	10	-987.5260

Table 11: Table containing the Bayesian Information Criteria Score for Each K value. From that, seven partitions can combine Fisher von Mises Distribution for the location parameter(Longitude and Latitude). We checked ten values of k from 1 to 10, where the optimal number of partitions is 6. This is done for the bird species *Black Bellied Plover*

Parameter	ρ Value
Solar Azimuth Change	0.3507626
Magnetic declination	0.3812564
Solar Zenith Change	0.4441466
Lunar Azimuth Change	0.2994615

Table 12: *gridwise*_{BlackBelliedPlover}

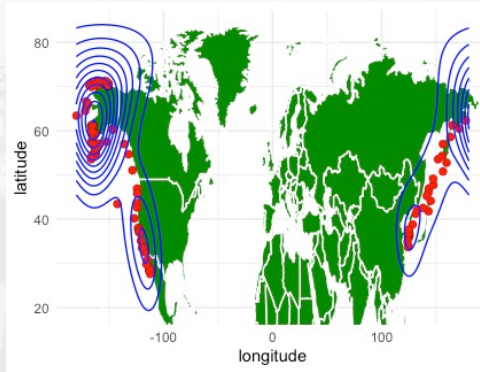
vonMises Fisher's Partitions Regression Results Black Bellied Plover

Part	Parameters used for regression with θ	ρ values(Circular Correlation values)	P-value
Part-1	Magnetic declination	0.4508	0.27779
	Solar Azimuth Change	0.4116	0.729
	Solar Zenith Change	0.4934	0.165
	Lunar Azimuth Change	0.4929	0.9219
Part-2	Magnetic declination	0.5277	0.06662
	Solar Azimuth Change	0.1118	0.1915
	Solar Zenith Change	0.6321	0.08149
	Lunar Azimuth Change	0.5254	0.2916
Part-3	Magnetic declination	0.8963	0.208
	Solar Azimuth Change	0.8531	0.2143
	Solar Zenith Change	0.8973	0.1926
	Lunar Azimuth Change	0.8517	0.8941
Part-4	Magnetic declination	0.4866	0.2875
	Solar Azimuth Change	0.3175	0.6243
	Solar Zenith Change	0.5375	0.3035
	Lunar Azimuth Change	0.3314	0.8409
Part-5	Magnetic declination	0.87005	0.1447
	Solar Azimuth Change	0.8405	0.6429
	Solar Zenith Change	0.87422	0.12912
	Lunar Azimuth Change	0.8318	0.2723
Part-6	Magnetic declination	0.87005	0.0447
	Solar Azimuth Change	0.8405	0.6429
	Solar Zenith Change	0.9397	0.0801
	Lunar Azimuth Change	0.957	0.042

Table 13: Based on our supervised partitions where all the parameters follow von Mises distribution, we performed the circular circular regression and the correlation test in the above table for *Black Bellied Plover*

Results for *Pacific Loon* Migration Patterns

- **Flight Direction Analysis:**
 - Flight direction follows a von Mises Distribution.
 - Regression (Table 16): Solar Azimuth change most significant .
 - Magnetic field change: second most significant. .
- **Spatial Distribution:**
 - von Mises-Fisher mixture model (Table 14), optimal number of mixtures is 5.
- **Correlation Analysis:**
 - Geomagnetic changes: $\rho = 0.39$ (Table 15).
 - Solar Azimuth change: $\rho = 0.37$ (Table 15).
- **Canonical correlation** (direction vs. physical parameters): 0.1552672.



von Mises-Fisher density plot over observed positions of *Pacific Loon*.

vonMises Fisher's Mixture model & Gridwise Partition based analysis for Pacific Loon

K	BIC Values	K	BIC Values
1	-147.4662	6	-1076.1542
2	-937.2215	7	-1075.0842
3	-1030.6938	8	-1080.8440
4	-1064.73	9	-1070.9659
5	-1091.3939	10	-1055.7243

Table 14: Table containing the Bayesian Information Criteria Score for Each K value. From that, seven partitions can combine Fisher von Mises Distribution for the location parameter(Longitude and Latitude). We checked ten values of k from 1 to 10, where the optimal number of partitions is 5. This is done for the bird species *Pacific Loon*

Parameter	ρ Value
Solar Azimuth Change	0.3775842
Magnetic declination	0.3926586
Solar Zenith Change	0.2518959
Lunar Azimuth Change	0.3060159

Table 15: *gridwise*_{PacificLoon}

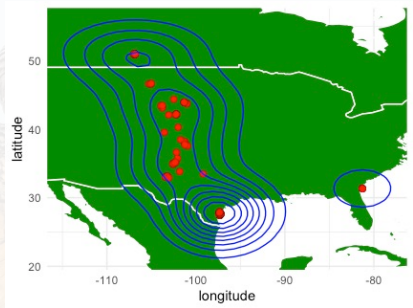
vonMises Fisher's Partitions Regression Results Pacific Loon

Part	Parameters used for regression with θ	ρ values	P-value
Part-1	Magnetic declination	0.4758	0.4497
	Solar Azimuth Change	0.463	0.1574
	Solar Zenith Change	0.3774	0.938
	Lunar Azimuth Change	0.4273	0.6588
Part-2	Magnetic declination	0.3928	0.314
	Solar Azimuth Change	0.3751	0.3444
	Solar Zenith Change	0.2474	0.9759
	Lunar Azimuth Change	0.3451	0.4005
Part-3	Magnetic declination	0.5883	0.1568
	Solar Azimuth Change	0.5705	0.430061
	Solar Zenith Change	0.4476	0.6033
	Lunar Azimuth Change	0.4775	0.5615
Part-4	Magnetic declination	0.3431	0.4702
	Solar Azimuth Change	0.5098	0.37305
	Solar Zenith Change	0.2118	0.4714
	Lunar Azimuth Change	0.2009	0.321
Part-5	Magnetic declination	0.4967	0.1307
	Solar Azimuth Change	0.5634	0.17411
	Solar Zenith Change	0.3599	0.9464
	Lunar Azimuth Change	0.4823	0.4013

Table 16: Based on our supervised partitions where all the parameters follow von Mises distribution, we performed the circular regression and the correlation test in the above table for *Pacific loon*

Results for *Long-billed Curlew* Migration Patterns

- **Flight Direction Analysis:**
 - Flight direction follows a von Mises Distribution.
 - Regression (Table 19): Magnetic field change most significant. .
- **Spatial Distribution:**
 - Based on von Mises-Fisher mixture model (Table 17), optimal number of mixtures is 2.
- **Correlation Analysis:**
 - Geomagnetic change: $\rho = 0.53$ (Table 18).
- **Canonical correlation** (direction vs. physical parameters): 0.2508563.



von Mises-Fisher density plot over observed positions of *Long-billed Curlew*.

vonMises Fisher's Mixture model & Gridwise Partition based analysis for Long-billed Curlew

K	BIC Values	K	BIC Values
1	-829.2649	6	-1167.5782
2	-1254.1596	7	-1196.8095
3	-1189.9874	8	-1191.6324
4	-1194.7899	9	-1165.8217
5	-1123.4830	10	-1176.4488

Table 17: Table containing the Bayesian Information Criteria Score for Each K value. From that, seven partitions can combine Fisher von Mises Distribution for the location parameter(Longitude and Latitude). We checked ten values of k from 1 to 10, where the optimal number of partitions is 2. This is done for the bird species *Long Bellied Curlew*

Parameter	ρ Value
Solar Zenith Change	0.2419564
Magnetic declination	0.5393819
Solar Azimuth Change	0.4256448
Lunar Azimuth Change	0.4092881

Table 18: *gridwise*_{Long-billedCurlew}

vonMises Fisher's Partitions Regression Results Long-billed Curlew

Part	Parameters used for regression with θ	ρ values	P-value
Part-1	Magnetic declination	0.2308	0.14659
	Solar Azimuth Change	0.1808	0.9774
	Solar Zenith Change	0.1674	0.994
	Lunar Azimuth Change	0.24776	0.79
Part-2	Magnetic declination	0.1744	0.1699
	Solar Azimuth Change	0.0189	0.7418
	Solar Zenith Change	0.02015	0.1962
	Lunar Azimuth Change	0.1619	0.1952

Table 19: Based on our supervised partitions where all the parameters follow von Mises distribution, we performed the circular circular regression and the correlation test in the above table for *Long Bellied curlew*

Key Conclusions on Bird Migration Patterns

- **Environmental Influence:** Geomagnetic and celestial cues (Sun, Moon, Earth's magnetic field) significantly impact bird migration, with reduced intensity during geomagnetic storms.
- **Correlation Findings:** Statistical tests confirm non-zero correlations between bird directional changes and environmental factors (magnetic inclination, declination, Sun-Moon positions).
- **Regional Patterns:**
 - South American bird locations follow a von Mises distribution.
 - North American locations: mixture of von Mises and circular uniform distributions.
- **Directional Analysis:** Haversine formula derives bird direction changes from consecutive positions.
- **Distribution Validation:** Bird species' directional changes statistically follow von Mises distributions.
- **Novel Model:** Circular-circular regression model shows significant relationships between bird direction and solar/lunar positional changes.
- **Species-Specific Insights:** Analysis per species reveals magnetic declination and Sun-Moon positions drive path changes (ρ values exceed significance).
- **Canonical Correlation:** Collective influence of magnetic and celestial factors on bird paths confirmed, with significant correlation values.

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Thank You

22References



S Rao Jammalamadaka A SenGupta.

Topics in Circular Statistics.

Volume 5. World Scientific Publishing Co. Pte. Ltd, 2001.



Prithwish Ghosh, Debashis Chatterjee, and Amlan Banerjee.

**On the directional nature of celestial object's fall on the earth
(Part 1: Distribution of fireball shower, meteor fall & crater
on earth's surface).**

Monthly Notices of the Royal Astronomical Society, page
stae1066, 04 2024.



Prithwish Ghosh, Banerjee Amlan Chatterjee D, and Das SS.

**Do magnetic murmurs guide birds? a directional statistical
investigation for influence of earth's magnetic field on bird
navigation.**

PLoS ONE, 2024.



Kanti V Mardia, Peter E Jupp, and KV Mardia.

Directional statistics, volume 2.

Wiley Online Library, 2000.



M Vihtakari.

**ggoceanmaps: plot data on oceanographic maps using
“ggplot2”. r package version 1.2. 6, 2022.**